The Finite Element Method for Problems in Physics

Coding Assignment 4

Consider a three-dimensional domain defined by $x_1 = [0, 1]$ m; $x_2 = [0, 1]$ m; $x_3 = [0, 0.1]$ m. Solve the steady state and transient heat conduction problems with the following boundary conditions and initial conditions. Use $\rho = 3.8151$ x 10^6 N.m^{-2}K^{-1} (specific heat per unit volume), $\kappa = 385$ watt.m^{-1}K^{-1}, where $\kappa_{ij} = \kappa\delta_{ij}$. Assume $j = 0$ watt.m^{-2} on all edges/surfaces where no temperature/flux conditions are specified. Use a mesh of 20 x 20 x 1 elements.

1. (Steady State problem): Boundary conditions $u = 300$ K along $x_1 = 0$ m and $u = 310$ K along $x_1 = 1$m.

2. (Transient problem): Boundary conditions $u = 300$ K along $x_1 = 0$ m, $u = 310$ K along $x_1 = 1$ m. Initial conditions $u = 300$ K for $x_1 < 0.5$ m and $u = 300 + 20 * (x_1 - 0.5)$ K for $x_1 \geq 0.5$ m.

Your code should solve both the steady state and transient heat conduction problems using linear basis functions. You should use the v-method to solve for the transient solution. Your code should be able to use Backward Euler, Forward Euler, and Mid-Point schemes. Your code should also calculate the L_2 norm of $(u_{steadystate} - u_{t_n})$ at a given time step.